Problem Statement: You wish to invest S$50,000. You have identified five investment opportunities. Each is an “all-or-nothing” investment: you must invest the full amount or not invest at all.

• Investment1 requires an investment of S$16,000 and has a present value (a time-discounted

value) of S$23,000.

• Investment 2 requires S$14,000 and has a present value of S$25,000.

• Investment 3 requires S$22,000 and has a present value of S$28,000.

• Investment 4 requires S$12,000 and has a present value of S$14,000 and

• Investment 5 requires S$38,000 and has a present value of S$49,000.

Into which investments should you place your money to maximize your total present value?

Classic Knapsack problem

Original Problem 1:

To get **initial bound** for this problem will solve **using greedy fractional Knapsack**, we will sort investment based on return/investment ratio.

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| --- | --- | --- | --- | --- |
| Ratios [1.4375 | 1.785714 | 1.272727 | 1.166667 | 1.289474] |

Include investment2 first then we have 36000 to invest = total return 25000

Then we include investment 1 then we left 20000 to invest = total return 48000

Then we include investment 4 because we can’t include investment 3 and 5 because their investment more than remaining amount to invest. i.e., 22000 and 38000 more than 20000

So initial bound = 25000+23000+14000 = 62000. i.e., x = [1,1,0,1,0]

1. We solve this using Branch and Bound

LP Relaxation 1:

LP

When we solve this **LP 1 Relaxation**, objective value **73789.47368,** with x= [1,1,0,0,0.5263157895] using LP1.mos

1. Now we will branch on x1 and will form **LP2 and LP3,** LP2 with x1=1 and LP3 with x1 =0.

LP

Solving LP2 will give objective 73789.47368, with x= [1,1,0,0,0.5263157895] using LP2.mos

LP

Solving LP3 will give objective 71421.05263 with x= [0,1,0,0, 0.9473684211] using LP3.mos

1. Now, we will branch on x5 from **LP2** and will form **LP4 and LP5,** LP4 with x5=1 and LP5 with x5 =0.

LP

Solving LP4 will give objective 0 with x= [1,0,0,0,1] using LP4.mos. It is less the initial bound, so it pruned.

LP

Solving LP5 will give objective 73454.54545with x= [1,1,0.909,0, 0] using LP5.mos

1. we will branch on x5 from **LP3** and will form **LP6 and LP7,** LP6 with x5=1 and LP7 with x5 =0.

LP

Solving LP6 will give objective 70428.57143 with x= [0,0. 8571428571,0,0, 1]

LP

Solving LP6 will give objective **67000** with x= [0,1,1,1,0] using LP7.mos which is Integer feasible and more than initial bound (62000). Now this is best bound for problem.

1. Now, we will branch on x3 from **LP5** and will form **LP8 and LP9,** LP8 with x3=1 and LP9 with x3 =0.

LP 8

Solving LP8 will give objective 73454.54545with x= [1,1,0.909,0,0], using LP8.mos

LP 9

Solving LP9 will give objective 65000 with x= [1,0,1,1, 0], using LP9.mos which is less than best bound 67000, so this node will be pruned.

1. Now, we will branch on x3 from **LP6** and will form **LP10 and LP11,** LP10 with x3=1 and LP11 with x3 =0.

LP 10

Solving LP10 will give objective 0 with x= [0,0,0,0,0], using LP10.mos, which is less than best bound so it will be pruned.

LP 11

Solving LP11 will give objective 64272.72 with x= [0,0,0.54545,0, 1] using LP11.mos, it will be also pruned because it is less than best bound so far.

So, we can only branch on LP8.

1. Now, we will branch on x3 from **LP8** and will form **LP12 and LP13,** LP12 with x3=1 and LP13 with x3 =0.

LP 12

Solving LP12 will give objective 0 with x= [0,0,0,0,0] using LP12.mos. It is less than best bound so will be pruned.

LP 13

Solving LP13 will give objective 62000 with x= [1,0,1,1, 0] using LP13.mos, which is less than 67000. So, we will prune this node also.

As we don’t, have any node to explore now. 67000 is optimal solution, with

To invest in following investment i.e., investment2, investment3 and investment 4.